Large N reduction with overlap fermions

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We revisit quenched reduction with fermions and explain how some old problems can be avoided using the overlap Dirac operator.

1. Introduction

This contribution, based on joint work, provides some background to Narayanan's talk [1].

Lattice field theory provides an approach to calculating numerical values or bounds for physical quantities from first principles without relying on perturbation theory or on non-systematic approximations. However, most of the numerical results about strong interactions involve fermions and are obtained in valence (quenched) QCD. This is not a systematic approximation because the zero flavor limit is singular. As a result, many lattice results still do not have a status higher than estimates coming from other phenomenological approximations that make ad-hoc approximations of different types. It makes little sense to go beyond few percent accuracy in simulations with valence QCD.

The long term goal of our work is to replace valence QCD with planar QCD in either the 't Hooft [2] or the Veneziano [3] limit. Unlike valence QCD, for many quantities of interest, planar QCD is believed to be the first term in a systemtic series. Our initial focus is on the two point functions of quark bilinears. In the planar limit, such a correlation function is characterized by an infinite number of pole masses and corresponding residues. Using the lattice, it should be possible to obtain good values for these correlation functions in momentum space at Euclidean momenta that are smaller than the inverse lattice spacing. Such information would be phenomenologically useful, for example, in the analysis of weak semi- and non-leptonic decays [4].

2. The planar limit

Let us consider QCD with gauge group $SU(N_c)$ instead of $SU(3)$. We are in Euclidean space, and assume that the volume V is already taken to infinity. Similarly, the ultraviolet cutoff Λ has also been eliminated. The coupling constant has disappeared too, generating an intrinsic scale in terms of which all predictions are pure numbers depending only on N_c. There are good choices for this intrinsic scale, in the sense that they lead to simple N_c dependences. Feynman diagrams and more general counting arguments tell us what powers of N_c to scale out so that remaining quantities have finite limits at $N_c = \infty$. In short, a nonperturbatively defined large N_c limit seems to exist and is reachable by lattice techniques.

In perturbation theory, the large N_c limit is easiest to understand before renormalization is taken into account. To get planar perturbative QCD one scales the gauge coupling constant g^2 by a factor of N_c and N_c is taken to infinity keeping $\lambda = g^2 N_c$ fixed. Using the known terms in the β-function one easily derives how a perturbative scale can be defined so that it stays finite at

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infinite N_c.

The above 't Hooft limit generalizes to the case where the number of flavors, N_f also is taken to infinity with the ratio $\rho = \frac{N_f}{N_c}$ kept fixed. This gives the Veneziano limit.

On the lattice, fast convergence to the $N_c = \infty$ limits was observed for the string tension, glueball mass, topological susceptibility, and finite temperature transition point in pure gauge theory [5]. These calculations are of theoretical interest; for practical applications it is certainly cheaper to just work at $N_c = 3$.

Having learned that the large N_c limit is meaningful outside perturbation theory and that it provides numbers quite close to $N_c = 3$, we note that large N_c reduction [6] provides a potential shortcut that would enable us to get the planar limit numbers in a substantially more efficient way because of the dramatic reduction in the number of degrees of freedom one needs to integrate over. In particular, the fermionic matrix becomes so small that it can be stored in its entirety in memory and dealt with in a more direct way than in standard simulations.

3. Quenched Reduction

Reduction works on lattices with finite numbers of sites. In physical applications, reduction implies a change in the order of limits of the traditional $\frac{1}{N_c}$ expansion. Now, we first take N_c (possibly also N_f) to infinity and only subsequently deal with the large V and large Λ limits.

The basic content of reduction is that there exists a certain "prototype" lattice gauge theory which encompasses lattice gauge theories with varying group sizes on differently sized lattices. However, the entire prototype family has the same planar limit. Thus, large V can be traded for larger N_c. The main point is that this tradeoff appears to be very lucrative.

The prototype variables are d $SU(N)$-matrices $T_\mu,\ \mu = 1, 2.., d$ and two fermionic $N \times M$ complex matrices $\bar{\Psi}$ and Ψ. The pure gauge action is given by

$$S_g = \frac{N}{4\lambda} \sum_{\mu,\nu} Tr \left(C_{\mu\nu} C_{\mu\nu}^\dagger \right), \quad C_{\mu\nu} \equiv [T_\mu, T_\nu]. \quad (1)$$

It is invariant under $SU(N)$ conjugation $T_\mu \to \Omega T_\mu \Omega^\dagger$ and under a $Z(N)^d$ acting by $T_\mu \to e^{\frac{2\pi\imath}{N} k_\mu} T_\mu$.

To add fermions one introduces a "Dirac" operator $D(T)$, which also transforms by conjugation, and a term $S_f = Tr \left(\bar{\Psi} D(T) \Psi \right)$ This breaks the $Z(N)^d$. There is an additional fermionic symmetry under the obvious action of $SU(M)$.

Assuming that N and M factorize, $N = nL_1 L_2 ... L_d$ and $M = mL_1 L_2 ... L_d$, one can constrain the basic variables in the prototype so that one obtains a traditional lattice gauge theory on a toroidal $L_1 \times L_2 .. \times L_d$ lattice. A planar limit can be defined on the lattice by scaling the lattice gauge coupling with n in the 't Hooft fashion. The basic claim of [6] was that for a certain class of interesting observables, the differently constained prototypes become identical in this planar limit, at finite lattice spacing. However when the continuum limit is approached, the identity is spoiled by a phase transition. Quenched reduction [7] is designed to eliminate this effect by applying another constraint to the prototype theory: The eigenvalues of the T_μ matrices are frozen at d independently drawn sets of uniformly distributed points on the unit circle parametrized by angles θ_μ^j. Further [8], the T_μ matrices entering $D(T)$ are multiplied by $U(1)$ phase factors of the form $e^{\imath p_\mu}$. Observables obtained by carrying out a quenched average with fixed angles have to be subsequently averaged over uniformly distributed θ_μ^j and p_μ. A large N limit can be defined as before, scaling the lattice coupling with N.

4. The resolution of some old problems

In the past, when chirality and the lattice were thought to be irreconcilable, the preferred way to introduce fermions was in a continuum version of quenched reduction [9]. In this version one replaces the unitary T_μ matrices of the prototype by hermitian matrices: $T_\mu \to 1 + \imath A_\mu$. a_μ^j, the eigenvalues of A_μ, are frozen and drawn from a uniform distribution over an interval $[-\frac{\Lambda}{2}, \frac{\Lambda}{2}]$. Similarly, the Dirac operator $D(T)$ gets replaced by a $D(A)$ and the random variable p_μ is introduced by replacing A by $A_\mu + q_\mu$ with q_μ randomly drawn again from $[-\frac{\Lambda}{2}, \frac{\Lambda}{2}]$.

In perturbation theory this looks fine. Surprisingly, there seems to be no clash between the UV regularization and gauge invariance even when the continuum theory would be chiral and anomalous. In addition, there seems to be no room in even $d = 2k$ for topological charge, since the natural object

$$\epsilon_{\mu_1\nu_1\mu_2\nu_2....\mu_k\nu_k} \atop Tr\,([A_{\mu_1}, A_{\nu_1}][A_{\mu_2}, A_{\nu_2}]....[A_{\mu_k}, A_{\nu_k}]) \tag{2}$$

vanishes identically.

In the past, it was unclear how this regulated continuum reduced model differed from the lattice version. It made some sense that topology would become a strange concept because the disappearance of spacetime took away the concept of a local topological density. Also, so long as fermions were dealt with in the 't Hooft limit, the issue of anomalies did not look pressing. Nevertheless, there was enough uncertainty surrounding this issue that no further work on fermions in reduced models was done. The lattice provided no attractive alternative.

The main new observation in this context is that choosing $D(T)$ as the overlap Dirac operator [10] frees the lattice version with fermions from all difficulties. The overlap also provides a definition of topological charge [11] in the pure gauge reduced model, opening the possibility to be consistent with recent studies of topology on the lattice at large N_c in traditional formulations. Moreover, topological obstructions that reflect anomalies are known to exist in ordinary lattice gauge theories already on finite lattices, as emphasized in [12]. These obstructions will also appear in the reduced model. Thus, so long as we stay on the lattice, quenched reduction will not eradicate anomalies.

5. Comments

Reduction seems to be closely related to the technique of "field orbifolding" [13]. For a pure $U(N_c)$ gauge theory, the constraints that turn the prototype model into a regular lattice gauge theory are equivalent to imposing invariance under a subgroup of the symmetry group of the prototype. It is known that field orbifolding preserves the planar limit in certain continuum filed theories and has a geometrical interpretation in string theory. Could one gain new insights into reduction from this?

Numerically, twisted reduction [14] used to be considered superior to quenching in the pure gauge sector. Unfortunately, the introduction of fermions is problematic [15]. Can one get around the fermion problem in some way while sticking to twisted reduction instead of quenched reduction in the pure gauge sector?

6. Conclusions

This is a good time to numerically revisit quenched reduction. Some of the old problems have been solved: Instanton effects and anomalies can be incorporated. Presently available computational power, albeit still short of what is needed for a full QCD simulation, might be amply capable of dealing with reduced models.

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